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THREE-DIMENSIONAL INTERNAL FLOWS IN TURBOMACHINERY

TASK 1 - SECONDARY FLOW IN DIFFUSING CASCADES

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SECONDARY FLOW IN DIFFUSING CASCADES

Research Objectives

a. Develop a three dimensional rotational flow analysis and a numerical solution procedure that can be used to determine the flow in the passages of a cascade with large deflection angles.

b. Construct a test rig to study the secondary flow phenomena in diffusion cascades with large deflections.

c. Obtain measurements of flow velocity, direction of flow, endwall static pressure and total pressure losses for a range of inlet Mach number representative of exit guide vanes.

d. Perform a parametric studies using the theoretical analysis and the experimental data to determine the effect of area diffusion on the secondary vorticity strength.

Status of the Research Efforts

1. The Analytical Work

i. The Problem Formulation:

The appropriate formulation for the equations governing the motion of the three dimensional inviscid rotational flow has been accomplished. The primary dependent variables in the formulation are the three flow velocity components and the through flow vorticity component.

The momentum equation is expressed as:

$$\bar{V} \times \bar{\Omega} = -\frac{1}{\rho} \nabla P \qquad (1)$$

where $\bar{V}$ is the velocity vector, P is the total pressure, ρ is the fluid density, and $\bar{\Omega}$ is the vorticity vector which is defined as:

$$\bar{\Omega} = \nabla \times \bar{V} \qquad (2)$$

For simplicity the solution is obtained for incompressible flow, in which case, the continuity equation is given by:

$$\nabla \cdot \bar{V} = 0 \qquad (3)$$

The momentum equation (1), is not the suitable form to be used since the pressure is not one of the dependent variables in the formulation. Therefore, the Helmholtz equation, which is given below, is used instead of equation (1),

$$\bar{V} \cdot \nabla\bar{\Omega} = \bar{\Omega} \cdot \nabla\bar{V} \qquad (4)$$

Equation (4) can be easily derived using equations (1), (2) and (3). Equations (2), (3) and (4) are the governing equations, and are solved for the three velocity components and the through vorticity component of the three dimensional rotational flow.

ii. <u>The Numerical Solution</u>:

The through flow velocity and through flow vorticity components are computed from Helmholtz equation using a marching technique. The secondary flow velocities on the other hand are determined from the simultaneous solution of the continuity equation and the through flow vorticity equation. The approach used in the numerical solution is iterative since the secondary flow velocities appear in the convective terms and the through flow velocity and vorticity contribute to the source and rotationality terms in the cross planes. These will be referred to as the outer iterations, to distinguish them from those used in the numerical solutions for the secondary velocities.

In order to develop and test the numerical solution without the unnecessary added complications of the leading edge vortex, a simple duct geometry was used, in order to be able to compare the results of the computations with the experimental data of reference 1, and also with the computational results of references 2 and 3.

A great part of the effort in developing the numerical methods have been devoted to the methods of computing the secondary velocities. The governing equations for the secondary velocity components are:

$$\frac{\partial}{\partial x_1}(x_1 u_1) + \frac{\partial}{\partial x_2}(x_1 u_2) = x_1 S \qquad (5)$$

$$\frac{\partial}{\partial x_2}(u_1) - \frac{\partial}{\partial x_1}(u_2) = -\omega \qquad (6)$$

Where u_1 and u_2 are the velocity components in the radial and axial directions (x_1, x_2) respectively, S is the source/sink term, and ω is the through flow vorticity component. Equation (5) represents the principal of conservation of mass when the source term, S, is expressed in terms of the through flow velocity gradient.

Two different approaches have been tried for the solution of the above equations. In the first approach, the secondary velocities were expressed in terms of a stream function ψ and a potential function ϕ as follows:

$$u_1 = \frac{\partial \phi}{\partial x_1} + \frac{1}{x_1} \frac{\partial \psi}{\partial x_2} \tag{7}$$

$$u_2 = \frac{\partial \phi}{\partial x_2} - \frac{1}{x_1} \frac{\partial \phi}{\partial x_1} \tag{8}$$

The governing equations for the potential and stream functions are obtained from the substitution of equations (7) and (8) into equations (5) and (6):

$$\frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2} + \frac{1}{x_1} \frac{\partial \phi}{\partial x_1} = S \tag{9}$$

and

$$\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_2^2} - \frac{1}{x_1} \frac{\partial \psi}{\partial x_1} = - x_1 \omega \tag{10}$$

With the following conditions over all of the boundaries:

$$\psi = 0 \tag{11}$$

$$\frac{\partial \phi}{\partial n} = 0 \tag{12}$$

where n is the direction perpendicular to the solid boundaries.

In the second approach, cross-differentiation was used to obtain two higher order equations in the secondary velocities u_1, u_2. By differentiating equation (5) with respect to x_1 and equation (6) with respect to x_2 and adding the results, one obtains:

$$\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} + \frac{1}{x_1} \frac{\partial u_1}{\partial x_1} - \frac{1}{x_1^2} u_1 = \frac{\partial S}{\partial x_1} - \frac{\partial \omega}{\partial x_2} \tag{13}$$

and by differentiating equation (5) with respect to x_2 and equation (6) with respect to x_1 and subtracting the results, one obtains:

$$\frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{1}{x_1}\frac{\partial u_2}{\partial x_1} = \frac{\partial S}{\partial x_2} + \frac{\partial \omega}{\partial x_1} + \frac{\omega}{x_1} \tag{14}$$

The velocity component, u_n, perpendicular to a solid boundary must vanish.

$$u_n = 0 \tag{15}$$

In this approach, the required additional boundary conditions for nonviscous flow are obtained from equation (6) written at the boundaries

$$\frac{\partial u_t}{\partial n} = \omega \tag{16}$$

where u_t is the velocity component in the direction tangent to the boundary.

It is clear from equations (9) and (10) and from equations (13) and (14) that in both approaches, the solution of two second order partial differential equations of the Poisson type is required. According to equations (11) and (12), the boundary conditions are of the Dirichlet type for the stream function ψ, and of the Neumann type for the potential function ϕ. In the second approach, according to equations (15) and (16), the boundary conditions for the secondary velocities are of the Neumann type over part of the boundary.

In the course of this investigation, difficulties were encountered in convergence of the numerical solution using both approaches. It was found that this was mainly a result of the inaccuracies in the calculations of the secondary velocities, caused by the Neumann type boundary conditions. Even with the special techniques for handling these boundary conditions in the numerical solution of Poisson equation (4), problems were still encountered in the convergence of both inner and outer iterations. At that stage, we went back and reexamined very carefully previously surveyed three dimensional flow computations and concluded that difficulties have always been encountered in the convergence of the inviscid flow solutions whenever the flow exhibited rotationality. A closer examination of the studies in references 2 and 3 revealed that those investigators introduced, under different kinds of justifications, what is equivalent to artificial sources and sinks to force the convergence of their solutions. We therefore decided to concentrate on developing an effective method for solving the continuity and the vorticity equations in order to avoid, if possible, the convergence problems associated with the resulting Poisson equations with Neumann boundary conditions.

In looking for a fresh approach, for computing the secondary velocities, we introduced a new concept in our analysis. Instead of using the stream and potential function, as in equations (7) and (8), a new type of function which will be referred to as "The streamlike function" was defined.

The Streamlike Function

The streamlike function is defined, such that the continuity equation (5) is identically satisfied. The secondary velocity components (u_1 and u_2) are expressed in terms of the streamlike function and the source term, S, as follows:

$$u_1 = \frac{1}{x_1} \frac{\partial \chi_1}{\partial x_2} + \frac{1}{x_1} \int x_1 \, S \, dx_1 \tag{17}$$

and

$$u_2 = - \frac{1}{x_1} \frac{\partial \chi_1}{\partial x_1} \tag{18}$$

The deviation from the standard definition of the stream function appears in the velocity component u_1, given by equation (17).

When equations (17) and (18) are substituted into equation (6), one obtains:

$$\frac{\partial^2 \chi_1}{\partial x_1^2} + \frac{\partial^2 \chi_1}{\partial x_2^2} - \frac{1}{x_1} \frac{\partial \chi_1}{\partial x_1} = \sigma_1 \tag{19}$$

where

$$\sigma_1 = x_1 \, \omega + \frac{\partial}{\partial x_2} \int x_1 \, S \, dx_1 \tag{20}$$

Since the continuity equation (5) is identically satisfied, the continuity and rotationality equations are actually transformed through the use of the streamlike function into a single Poisson equation (19).

The condition of zero normal velocity component at the stationary solid boundaries, is expressed in terms of the streamlike function, χ_1, as:

$$\chi_1 = - \iint x_1 \, S \, dx_1 \, dx_2 \tag{21}$$

It is clear from equation (21) that the boundary conditions for the streamlike function are of the Dirichlet type. The novelty of the new approach lies in the fact that instead of having to solve two Poisson equations at each cross plane, one now has to solve a single equation in the streamlike function. Another advantage of the new streamlike function is the Dirichlet boundary conditions over all the boundaries.

One should mention here that a different streamlike function, χ_2, could also be defined to accomplish the same task. The deviation from the traditional stream function definition can be included in the u_2 velocity component as follows:

$$u_1 = \frac{1}{x_1} \frac{\partial \chi_2}{\partial x_2} \tag{22}$$

and

$$u_2 = - \frac{1}{x_1} \frac{\partial \chi_2}{\partial x_1} + \frac{1}{x_1} \int x_1 \, S \, dx_2 \tag{23}$$

The streamlike function χ_2 also satisfies equation (5) identically. When the above two equations are substituted into equation (8), one obtains:

$$\frac{\partial^2 \chi_2}{\partial x_1^2} + \frac{\partial^2 \chi_2}{\partial x_2^2} - \frac{1}{x_1} \frac{\partial \chi_2}{\partial x_1} = - \sigma_2 \tag{24}$$

where

$$\sigma_2 = x_1 \omega - x_1 \frac{\partial}{\partial x_1} \int S \, dx_2 \tag{25}$$

A single second order equation in the streamlike function χ_2 results also in this case, with Dirichlet boundary conditions. The details of this new approach, as well as examples of its applications in different flow problems has been published (Ref. 5) as AIAA Paper No. 79-146.

The formulation of the problem in terms of the new streamlike function is expected to result in saving computer time and storage. One should need fewer iterations to solve the single Poisson equation with Dirichlet boundary conditions. In addition, we believe that the presence of the source integral over the passage cross-sectional area in the boundary conditions (Eq. 21) could even accelerate the convergence of the solution. For all these reasons, the new formulation is believed to be very important in achieving the objective of the analytical numerical investigation. We are presently incorporating this formulation into the numerical solution procedure. The primary results obtained so far leads us to believe that the original problems in the convergence of the numerical solution have been overcome.

2. The Experimental Work

We are now in the process of building our tunnel for the experimental flow measurements. Most of the experimental efforts have been directed up till now at acquiring through NSF sponsorship a Laser Doppler Velocimeter (LDV) system which will be used in the experimental measurements. This system consists of an argon-ion laser (Spectra Physics Model 164-09), a back scatter type optical system, an oscilloscope, a signal processing system (Thermo Systems, Inc., Model 1990), and a pdp-8e computer. The laser and all the optical components have been mounted on a traversing mechanism in order to be able to survey the tunnel test sections. The laser-doppler technique has been tested in a simpler flow field of another tunnel. Aluminum oxide powder was used as the seeding material because of the relative ease with which it can be introduced into the flow. The accuracy of the velocity measurements have been tested and the data was found to be reproducible and precise enough to justify confidence in the experimental system. This LDV system will be very useful in our measurements since flow disturbances caused by the presence of probes in this highly complicated three dimensional flow field will be avoided.

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5. Hamed, A. and Abdallah, S., "A New Approach for Solving the Vorticity and Continuity Equations in Turbomachinery Ducts," AIAA Paper No. 79-0046, 1979.

List of Written Publications

A. Hamed, and S. Abdallah, "A New Approach for Solving the Vorticity and Continuity Equations in Turbomachinery Ducts," AIAA Paper No. 79-0046, 1979.

A. Hamed, and S. Abdallah, "Solution of the Nonhomogeneous Cauchy Reiman Equations," under preparation to be submitted for publication in the Journal of Aircraft.

List of Professional Personnel

Associated with the Research Effort

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Paper Presentations

"The Streamlike Function, A New Approach in Solving the Vorticity and Continuity Equations," to be presented at the AIAA Mini Symposium, Wright Patterson Air Force Base, March 21, 1979.

"A New Approach in Solving the Vorticity and Continuity Equations in Turbomachinery Ducts," presented at the AIAA 17th Aerospace Sciences Meeting, New Orleans, January 15, 1979.

"Three-Dimensional Rotational Flow in Highly Curved Ducts Due to Inlet Vorticity," presented at the Symposium on Aerospace Systems Technology, Present and Future, Wright Patterson Air Force Base, March 23, 1978.

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The nonviscous three-dimensional rotational flow is analyzed in highly curved channels. The problem is formulated in terms of the velocity and vorticity. The secondary velocities are computed for the continuity and vorticity equations, which are nonhomogeneous first order equations. Previous formulations of these equations resulted in two second order differential equations. A new approach is developed, which is applicable in (cont'd)

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a generalized two dimensional domain or axisymmetric field. It is based on the definition of a streamlike function which is used to transform these nonhomogeneous first order partial differential equations to a single second order equation with Dirichlet boundary conditions over the solid boundaries. The new approach is superior in terms of computational time and storage requirements.